Probabilistic Methods in Combinatorics

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Assignment 2

To solve for the Example class on 4th March. Submit the solution of Problem 2 by Sunday 2nd March if you wish feedback on it.

The solution of each problem should be no longer than one page!

Starred problems are typically harder. Don't worry if you cannot solve them.

Problem 1. Suppose that $n \geq 4$ and let H be an n-uniform hypergraph with at most $4^{n-1}/3^n$ edges. Prove that there is a colouring of the vertices of H by four colours so that in every edge all four colours are represented.

Problem 2. Let $G = (V, E)$ be a bipartite graph with n vertices and a list $S(v)$ of more than $\log_2 n$ colors associated with each vertex $v \in V$. Prove that there is a proper coloring of G assigning to each vertex v a colour from its list $S(v)$.

Problem 3*. Let $G = (V, E)$ be a graph on $n \geq 10$ vertices and suppose that if we add to G any edge not in G then the number of copies of a complete graph on 10 vertices in it increases. Show that the number of edges of G is at least $8n - 36$.

Problem 4*. Let m, n and t be positive integers such that $n > t \geq 0$. For a vector $x \in \mathbb{R}^n$, we denote by $w(x)$ the number of nonzero coordinates of x.

Suppose we are given vectors $a_1, a_2, \ldots, a_m, b_1, b_2, \ldots, b_m \in \{0,1\}^n$ such that $w(a_i - b_i) = t+1$ for all $i \in \{1, 2, \ldots, m\}$ and $w(a_i - b_j) \leq t$ for all $i \neq j$. Then, show that $m \leq 2^{t+1}$.